**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans) 0.2676

from scipy import stats

from scipy.stats import norm

# Find z score at x = 50

z = (50-45)/8

z

# Find probability for p(x>50)

stats.norm.cdf(0.625)

1 - stats.norm.cdf(0.625)

p\_value = stats.norm.cdf(0.625)

1 - p\_value

0.26598552904870054

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

FALSE:

# p(x>44) Employees older than 44 years of age

stats.norm.cdf(44,38,6)

1 - stats.norm.cdf(44,38,6)

0.15865525393145707

# p(38<x<44) Employees between 38 and 44 years of age

stats.norm.cdf(44,38,6) - stats.norm.cdf(38,38,6)

0.3413447460685429

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

TRUE

1. #b
2. # p(x<30) Employees under 30 years of age
3. stats.norm.cdf(30,38,6)

0.09121121972586788

# Number of employees attending training under the age of 30 years

400\* stats.norm.cdf(30,38,6)

36.484487890347154

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans) The mean of both distributions is the same, but the variance of 2X1 is twice the variance of X1 + X2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans) D

#4

stats.norm.interval(0.99,100,20)

(48.48341392902199, 151.516586070978)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans) import numpy as np

# Mean profit from two different divisions of a company

mean = 5+7

print('Mean profit is', mean\*45, 'Million')

Mean profit is 540 Million

# Variance of profits from two different divisions of company

sd = np.sqrt((9) + (16))

print('Standard deviation is', sd\*45, 'Million')

Standard deviation is 225.0 Million

#A

stats.norm.interval(0.95,540,225)

(99.00810347848784, 980.9918965215122)

#b

# To compute 5th Percentile, we use the formula X= μ + Zσ; wherein from z table, 5 percentile = -1.645

x = 540 + (-1.645)\*(225)

x

169.875

#c

# Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3)

0.0477903522728147

# Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4)

0.040059156863817086